Statistical analysis of Leonardo´s rule

Leonardo da Vinci observed in his notebooks that ‘‘all the branches of a tree at every stage of its height when put together are equal in thickness to the trunk’’ [1], which means that when a mother branch of diameter d splits into N daughter branches of diameters d_i , the following relation holds on average

$$d^{\Delta} = \sum_{i=1}^{N} d_i^{\Delta}$$

where the Leonardo exponent is ≈ 2 . Surprisingly, there have been few assessments of this rule.

In [2] it is proposed that Leonardo's rule is a consequence of the self-similarity of the tree trunk and wind-induced stress. In the mentioned paper some ad hoc hypothesis, based in fractal theory and fracture theory, were introduced in order to obtain Leonardo's law.
However, it is curious that Leonardo himself proposes a very simple ex-planation of this rule based on the characteristics of uid motion. \When a branch grows, Leonardo argues, its thickness will depend on the amount of sap it receives from the one below the branching point. In the tree as a whole, there is a constant flow of sap, which rises up through the trunk and divides between the branches owing through succesive ramications.

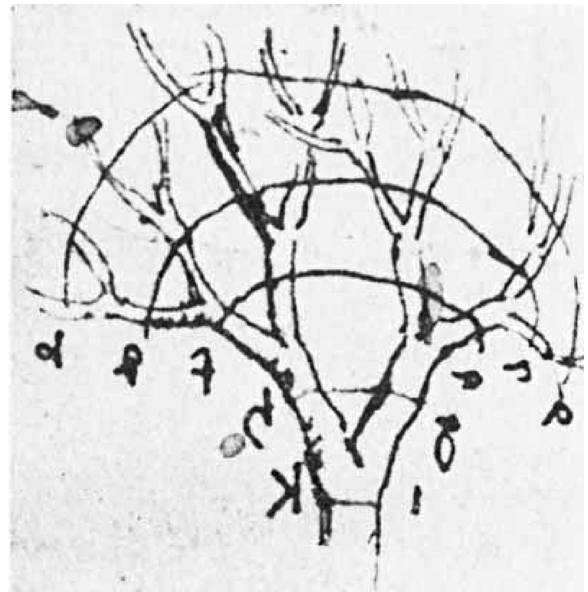
the total quantity of sap carried by the tree is constant, the quantity carried by each branch will be proportional to its cross section, so the total cross section at each level will be equal to that of a trunk" [3].
This argument is naive, since the flux of the sap, as viscous liquid, is not proportional to the cross section of the branch, but to the fourth power of its radius, if we consider branches and trunk as cylinders.
In our opinion, though Leonardo's rule is not a trivial result from uid mechanics, a simple and direct approach, based on mass conservation, can be made to explain this observation.
The amount of fluid that goes through a conduit of this type is given by the Poiseuille's Law

$$Q(R) = \frac{\pi \nabla P}{8\eta} R^4$$

Q is the flux

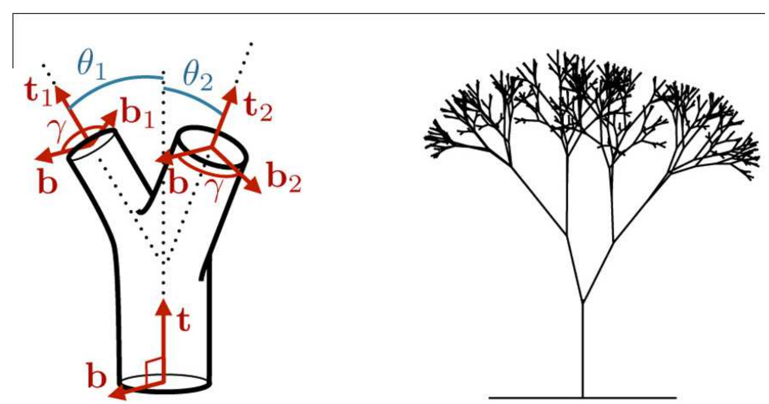
Da Vinci: Codex Atlanticus

$$\boxed{r_1^2 + r_2^2 = r^2}$$



C. Eloy, Phys. Rev. Lett, 107, 258101, (2011)

Criterion: resistance to fracture



Let´s apply the criterion of Leonardo, but considering the viscosity of sap and that the flux goes through a distribution of vessels which obey a Lévi distribution of radius. The origin of this distribution will be justified in the next pages.

If we calculate the total flux Q on a given branch considering that it is distributed among a network of “pipes” Levi-distributed by radius, the total flux of sap transported by the branch is the sum of all contributions of the vessels.

$$Q(R) = \int_0^R f(r) Q(r) dr$$

$$Q(r) \sim r^4$$

$$f(r) \sim r^{-x}$$

(asymptotic behavior)

Then, to fulfill Leonardo´s rule x must be:

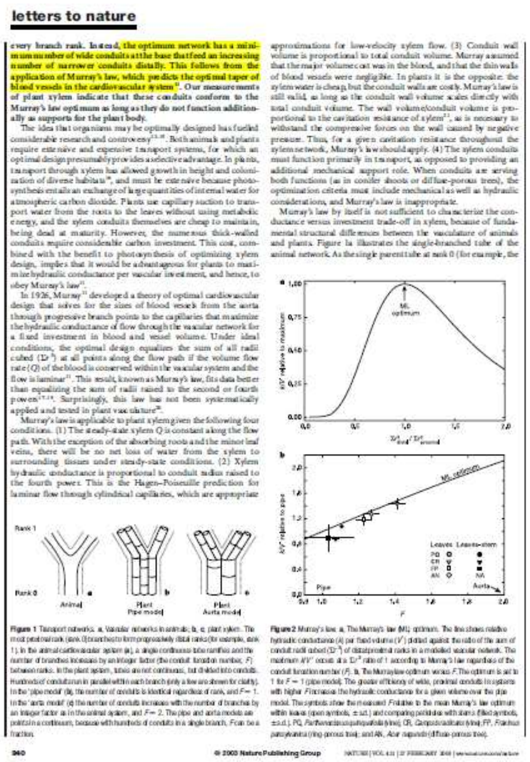
$$\boxed{x = 3}$$

How to justify this?

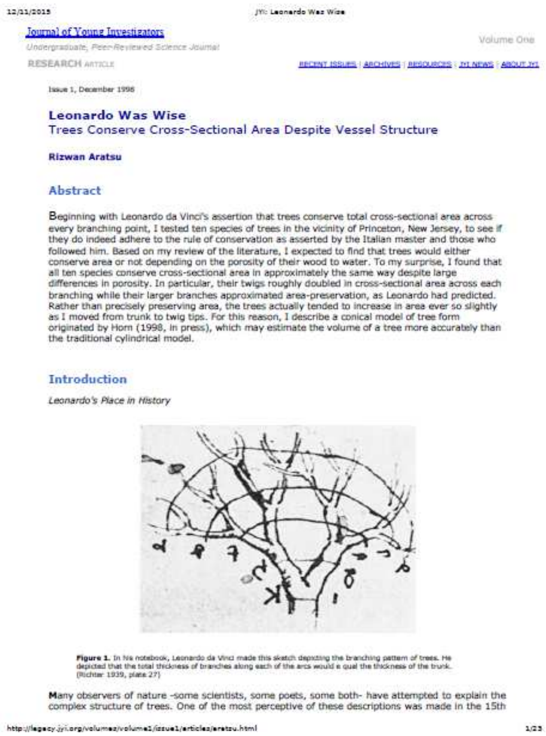
Drawing a Tree by Leonardo da V... *by corbisimages*

Zazzle

Papers of Murray and Christophe



letters to nature



12/11/2015 JYI: Leonardo Was Wise

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Leonardo Was Wise

Trees Conserve Cross-Sectional Area Despite Vessel Structure

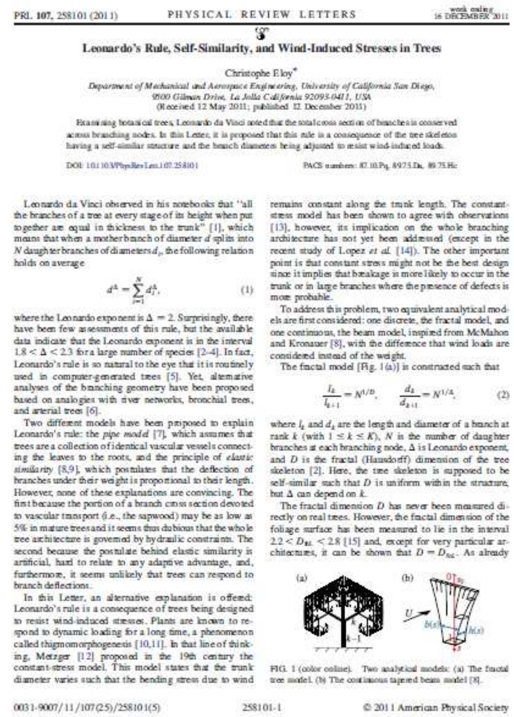
Rizwan Aratsu

Abstract

Beginning with Leonardo da Vinci's assertion that trees conserve total cross-sectional area across every branching point, I tested ten species of trees in the vicinity of Princeton, New Jersey, to see if they do indeed adhere to the rule of conservation as asserted by the Italian master and those who followed him. Based on my review of the literature, I expected to find that trees would either conserve area or not depending on the porosity of their wood to water. To my surprise, I found that all ten species conserve cross-sectional area in approximately the same way despite large differences in porosity. In particular, their twigs roughly doubled in cross-sectional area across each branching while their larger branches approximated area-preservation, as Leonardo had predicted. Rather than precisely preserving area, the trees actually tended to increase in area ever so slightly as I moved from trunk to twig tips. For this reason, I describe a conical model of tree form originated by Horn (1998, in press), which may estimate the volume of a tree more accurately than the traditional cylindrical model.

Introduction

Leonardo's Place in History



PRL 107, 258101 (2011) PHYSICAL REVIEW LETTERS week ending 16 DECEMBER 2011

Leonardo's Rule, Self-Similarity, and Wind-Induced Stresses in Trees

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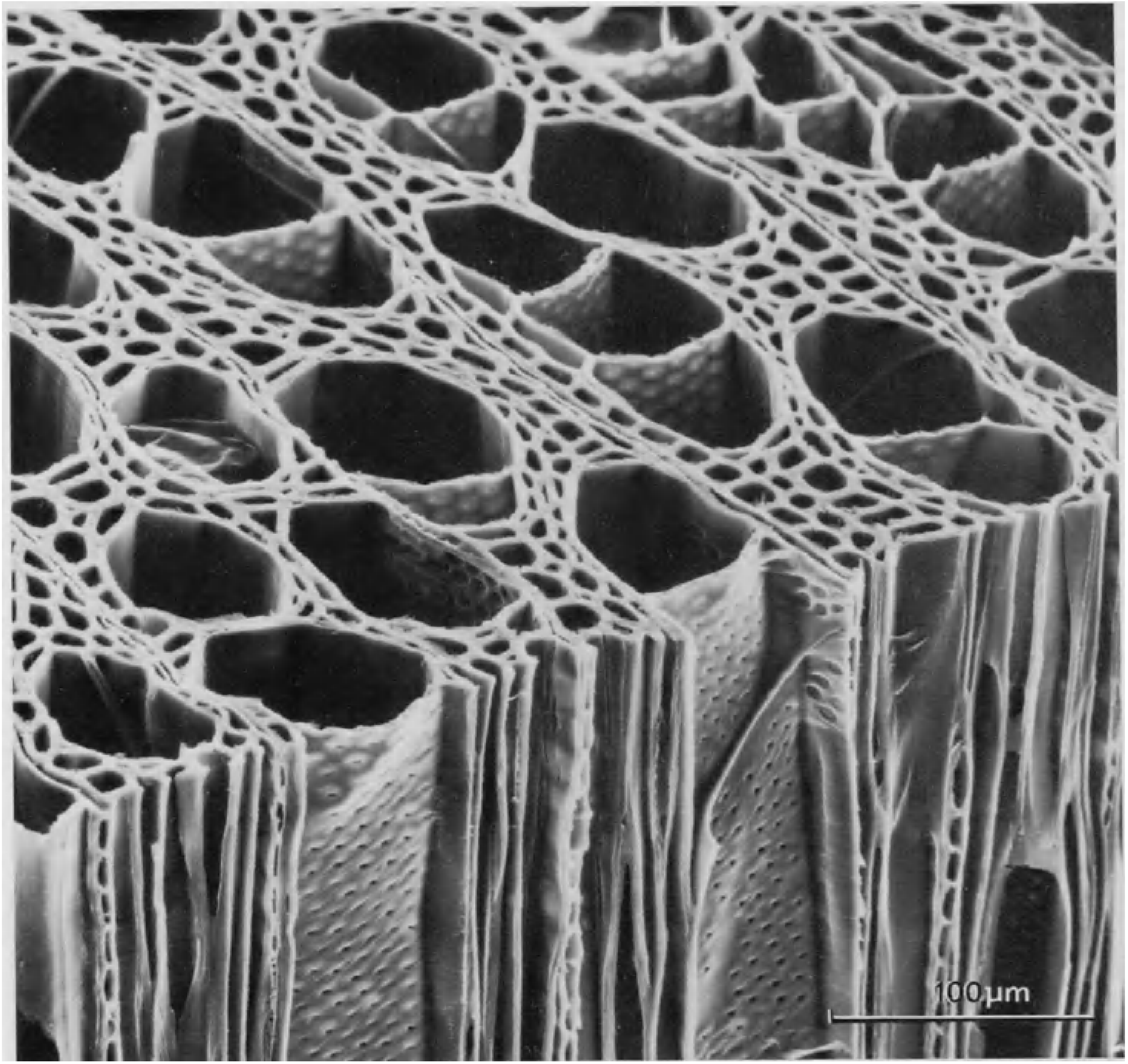
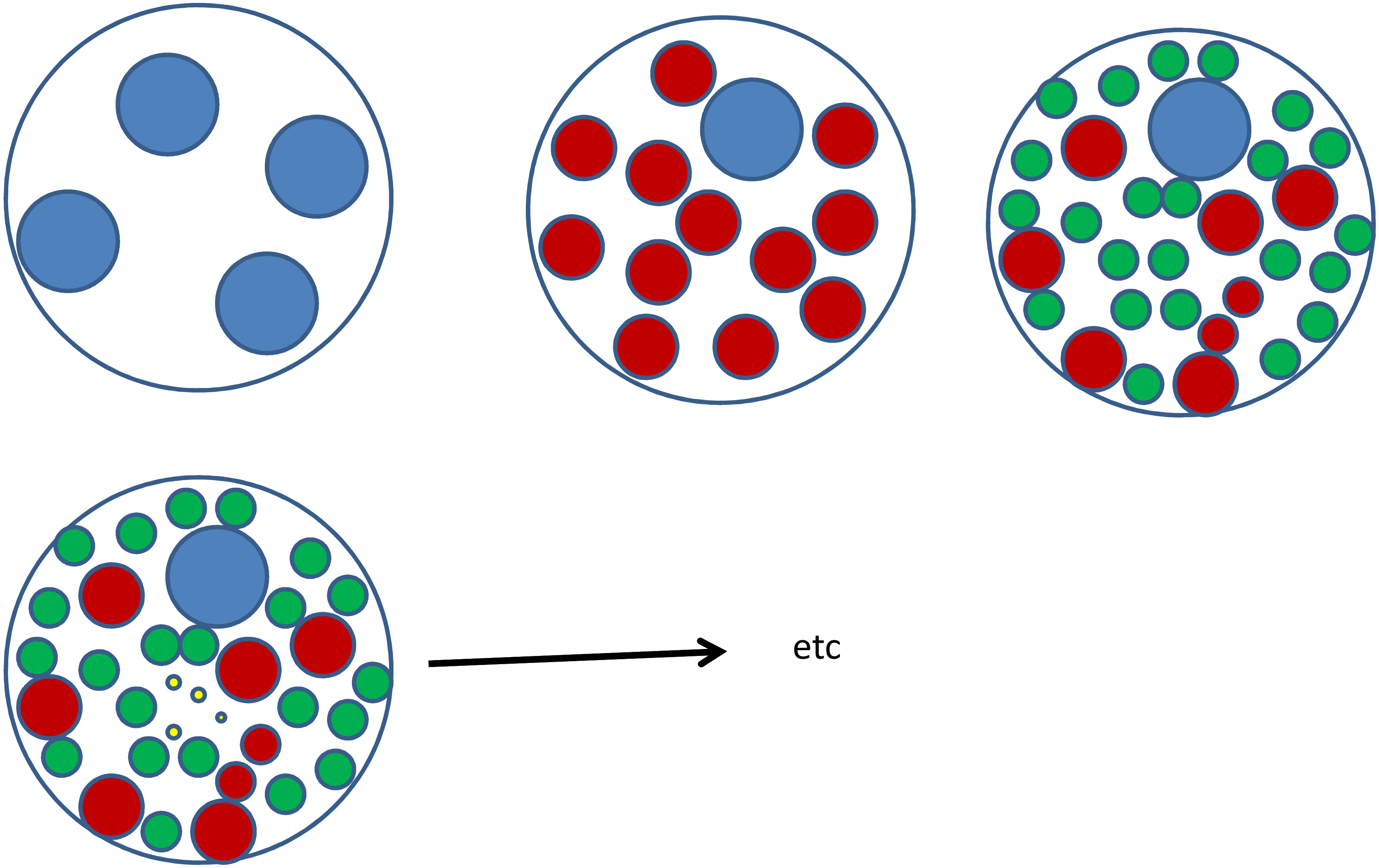
(Received 12 May 2011; published 12 December 2011)

Examining botanical trees, Leonardo da Vinci noted that the total cross section of branches is conserved across branching nodes. In this Letter, it is proposed that this rule is a consequence of the tree skeleton having a self-similar structure and the branch diameters being adjusted to resist wind-induced loads.

DOI: 10.1103/PhysRevLett.107.258101 PACS numbers: 87.10.Pq, 89.75.Da, 89.75.Hc


100µm

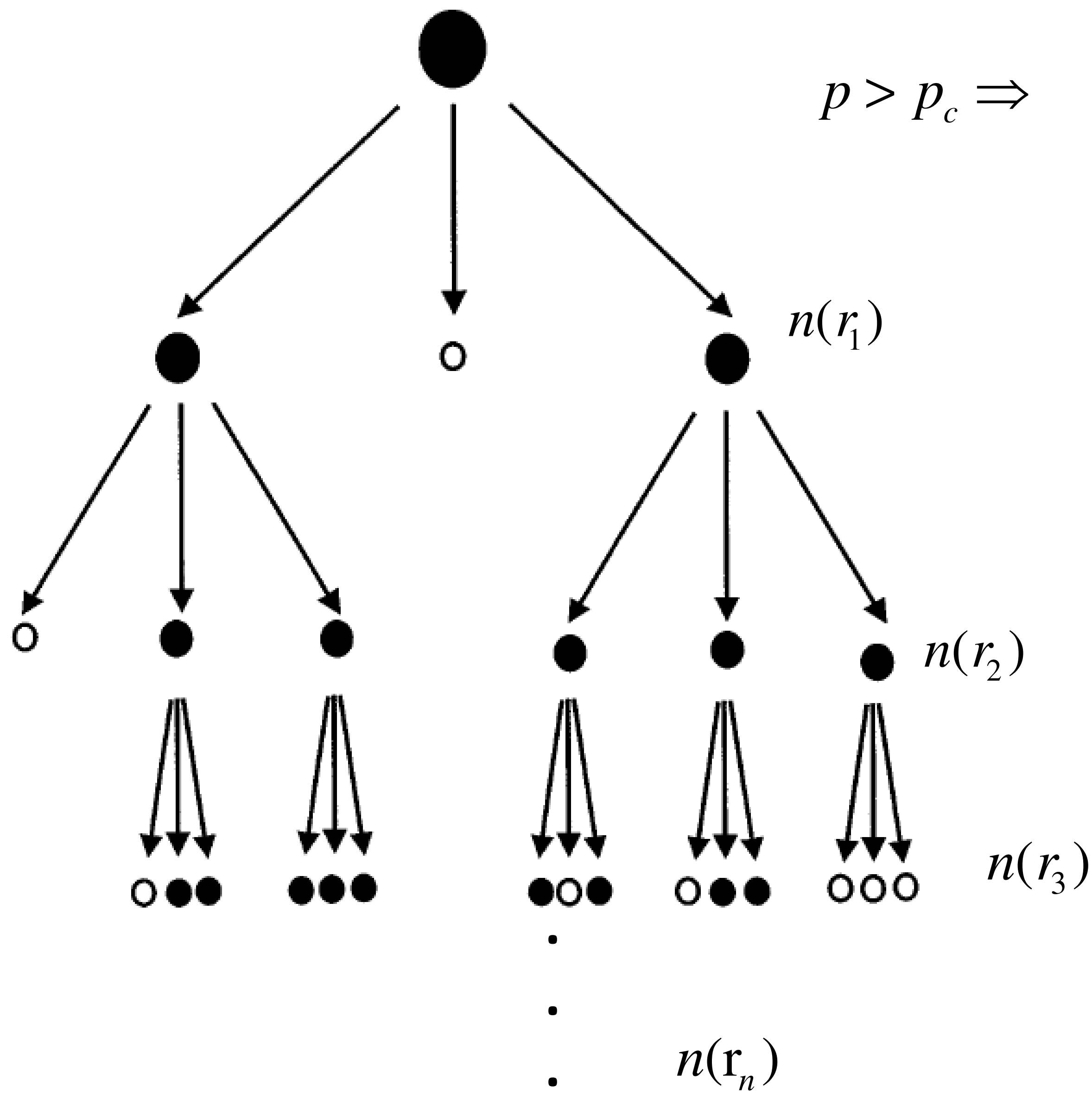
The previous photo of the wall between two vessels suggests a formation mechanism of evolutive type, i.e., a few big vessels bifurcate in smaller ones with a given probability, Then the new generation repeates the process, and so on, giving as a result a distribution of vessels broadly distributed, schematically illustrated as follows:


etc

This process suggests a representation in terms of a Bethe lattice, where
old vessels generate new ones with a given probability , thus dissappearing, so
that the problem becomes that of the percolation on a Bethe lattice.
This is a well known problem, where the critical probabiity to percolate an infinite
branch is known. In our case, let us consider that branching occurs conserving the flux, i.e.,
The new vessels conduct the same flux as the old one, progenitor.

Bethe

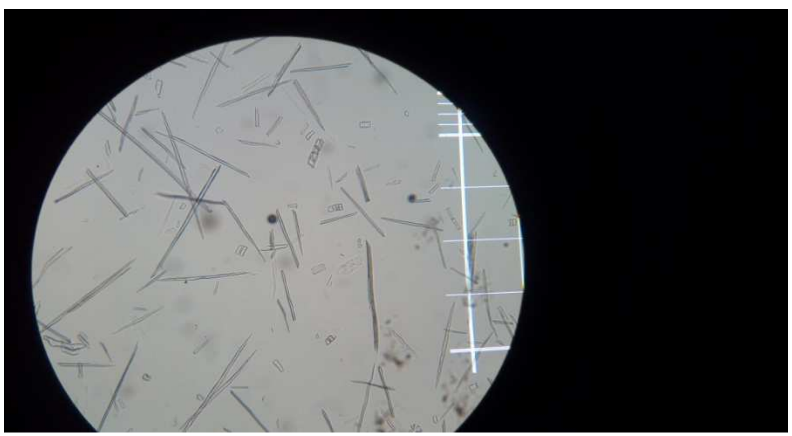
$p > p_c \Rightarrow$ Multiple scales

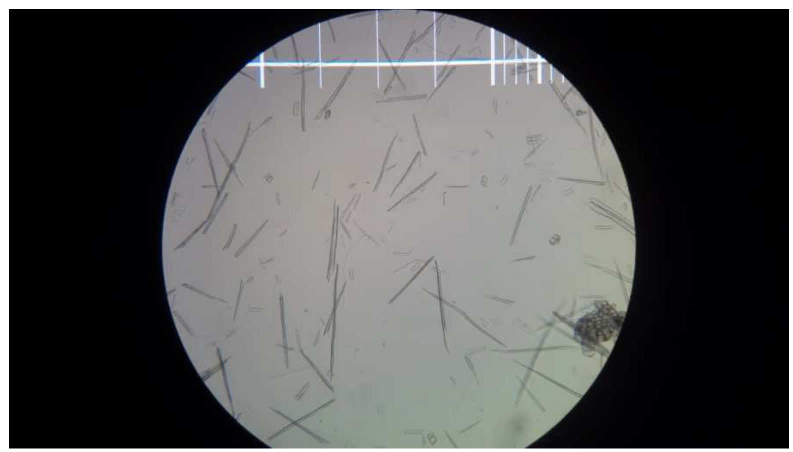


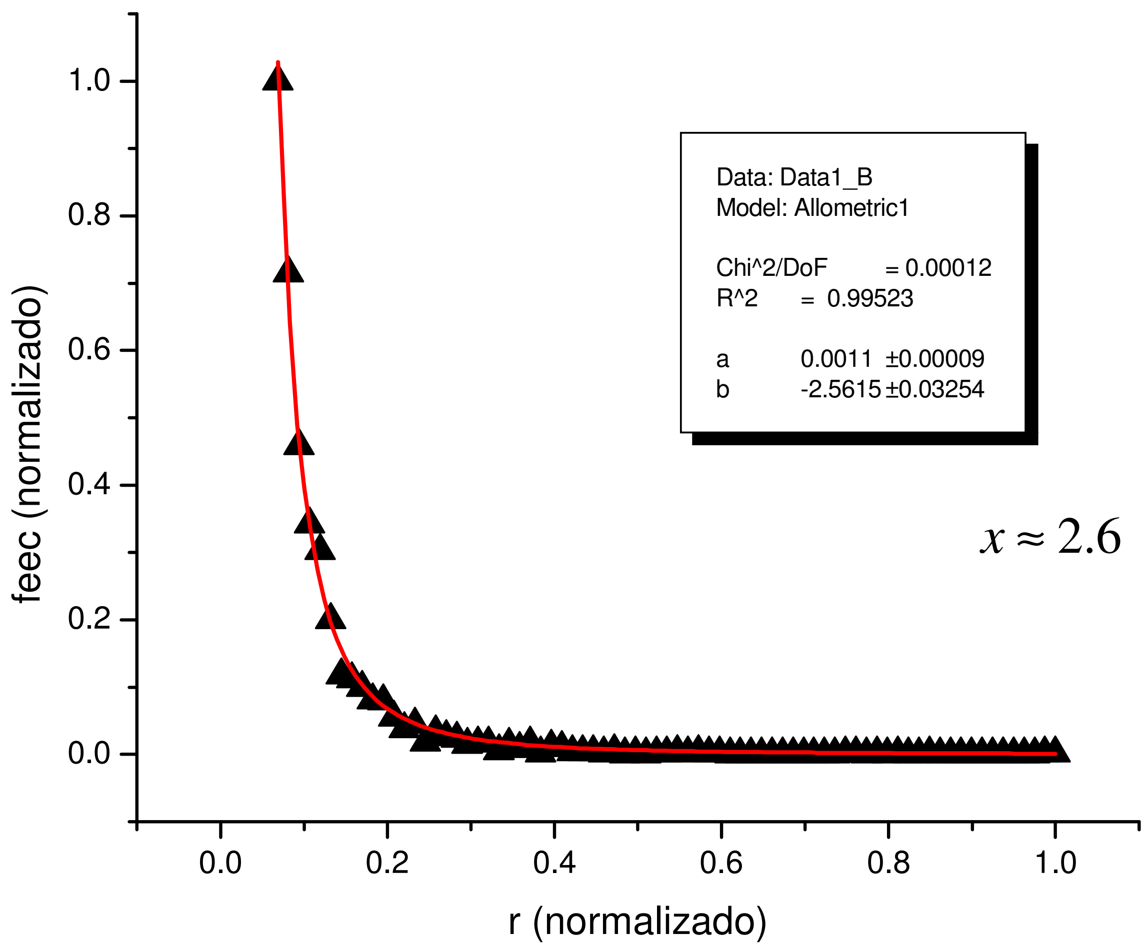
$$\Rightarrow \exists f(r)$$

$$p > p_c \Rightarrow f(r) \sim r^{-x}$$ No characterístic scale

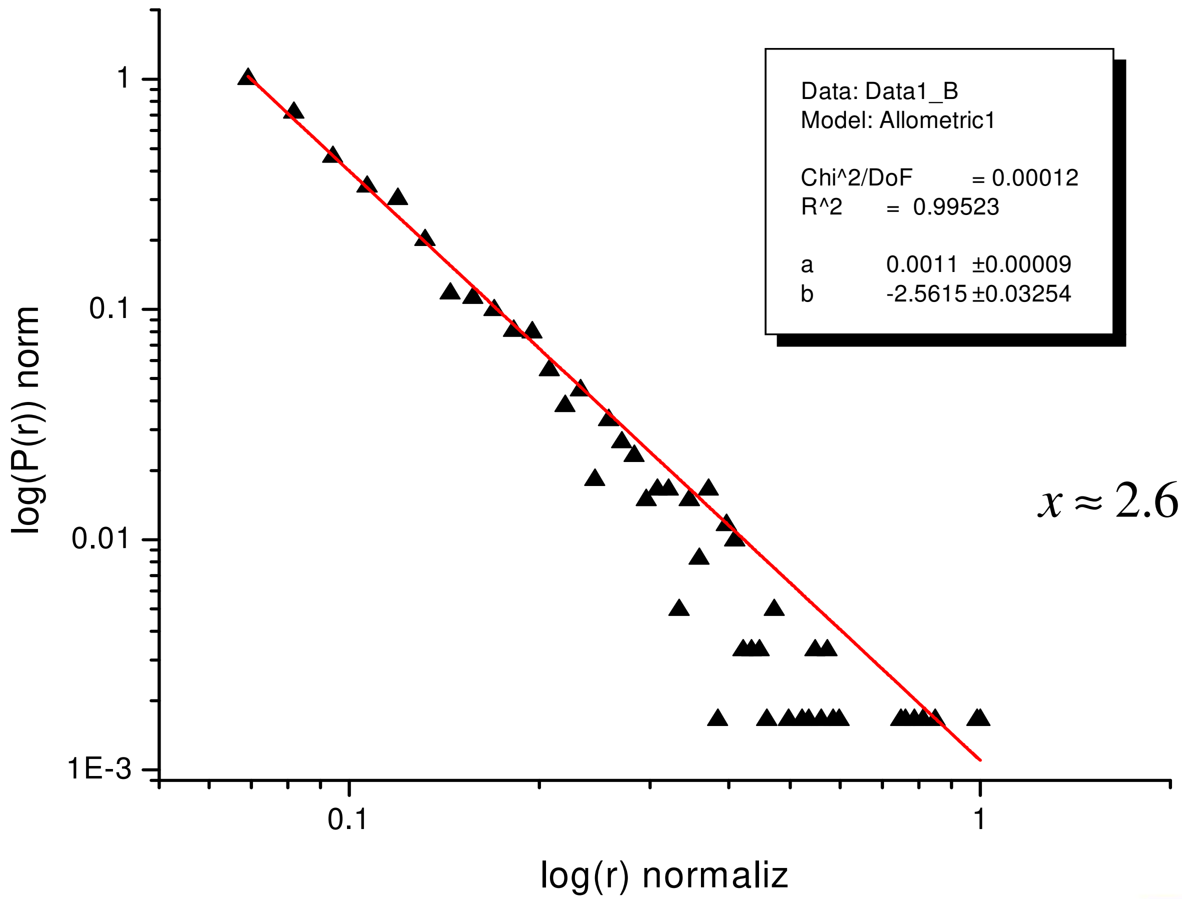
We tested this result (Levy distribution?)






Data: Data1_B
Model: Allometric1
Chi^2/DoF = 0.00012
R^2 = 0.99523
a 0.0011 ±0.00009
b -2.5615 ±0.03254
$x \approx 2.6$
feec (normalizado)
r (normalizado)
0.0
0.2
0.4
0.6
0.8
1.0

log-log:



$x \approx 2.6$

Well, is scaling, asymptotics of Levy!

$$Q(R) = \int_0^R f(r) Q(r) dr$$ as $$f(r) \sim r^{-x}$$

$$Q(r) \sim r^4$$

$$R^{\Delta} = R_1^{\Delta} + R_2^{\Delta}$$

Direct measure of branches (Capulin)

$$\Delta_{\exp} \in (1.82,\ 1.96) \qquad \Delta \approx 1.9$$

Direct measure of vessels: $x \approx 2.6 \pm 0.03 \quad \Rightarrow x \in (2.53,\ 2.59)$

$$\Rightarrow \Delta_{vasos} \in (2.41,\ 2.47)$$

Error in 0.45 (18%)!!

CONCLUSIONS

1- resistance to wind stress and nurture are both important criteria in Leonardo´s rule
2- vessels are Levy distributed, though fitting of critical exponent with experiment is not as good as we expected.
3.- Bethe lattice model must be improved to obtain a better fitted critical exponent

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2 Christophe Eloy : Leonardo’s Rule, Self-Similarity, andWind-Induced Stresses in Trees PRL 107, 258101 (2011)

3 Sotolongo-Costa, O., Gaggero-Sager, L. M., Oseguera-Manzanilla, T., and Daz-Guerrero, D. S. Comment concerning Leonardo's rule. arXiv:1608.06598v1 [physics.bio-ph] 23 Aug 2016